Test 3C - MTH 1210

Dr. Graham-Squire, Fall 2013

Name: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation! A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are not allowed on this test. Calculators are allowed on the first 4 questions of the test, however you should still show all of your work. No calculators are allowed on the last 6 questions.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Make sure you sign the pledge.
8. Number of questions = 10. Total Points = 80.

1. (12 points) The population of Fire-bellied Newts in the smaller of the Twin Lakes on Orcas island behaves according to the function

$$p(t) = \frac{5600}{2 + 26e^{-0.044t}}$$

where t is measured in years.

(a) What is the population initially?

(b) What is the population in the long run (as t goes to infinity)? Explain your reasoning.

(c) How many fire-bellied newts are there in the lake when t=10? Round to the nearest newt

(d) When does the population reach 1000? Round to nearest year

$e^0 = 1$

(a) $t=0 \Rightarrow p(0) = \frac{5600}{2+26e^{(-0.044)(0)}} = \frac{5600}{2+26e^0}$

$= \frac{5600}{28} = \boxed{200 \text{ newts}}$ 3

(b) As $t \to \infty$, $e^{-0.044t} = e^{-\infty} = \frac{1}{e^\infty} = 0$

$\Rightarrow$ as $t \to \infty$, $p(t) \to \frac{5600}{2+26(0)} = \frac{5600}{2} = \boxed{2800 \text{ newts}}$ 3

(c) $p(10) = \frac{5600}{2+26e^{-0.044(10)}} = 298.7 \approx \boxed{299 \text{ newts}}$ 2

(d) $1000 = \frac{5600}{2+26e^{-0.044t}}$ ✓

$\Rightarrow (2+26e^{-0.044t})1000 = 5600$ ✓

$\Rightarrow 2000 + 26000e^{-0.044t} = 5600$

$-2000 \quad -2000$

$\frac{26000e^{-0.044t}}{26000} = \frac{3600}{26000}$

$e^{-0.044t} = \frac{3600}{26000}$

$\Rightarrow \frac{\ln(\frac{3600}{26000})}{-0.044} = t$

$t \approx 44.9$ ✓

$\Rightarrow \boxed{t = 45 \text{ years}}$ 4

2. (10 points) Use laws of logarithms to expand and simplify the expressions as much as possible.

(a) $\ln\left(\frac{x^2y^5}{e^3}\right)$ $= \ln(x^2y^5) - \ln e^3 = \ln x^2 + \ln y^5 - 3(\ln e)$

$= \boxed{2\ln x + 5\ln y - 3}$

(b) $\log_2 \sqrt{x\sqrt{y}}$

$= \log_2 (xy^{1/2})^{1/2}$

$= \frac{1}{2}\log_2 (xy^{1/2})$

$= \frac{1}{2}(\log_2 x + \log_2 y^{1/2})$

$= \boxed{\frac{1}{2}(\log_2 x + \frac{1}{2}\log_2 y)}$

or $\frac{1}{2}\log_2 x + \frac{1}{4}\log_2 y$

3. (10 points) The rabbit population feeding off of Eddie's garden was 10 rabbits in the year 2010. In 2013, the number had grown to 20. Assuming the rabbit population fits an exponential growth model, in what year will the population reach 1000 rabbits? Round your answer to the nearest year.

use an exponential growth model to

Let 2010 be $t=0 \Rightarrow P=10$ ✓✓ $A=Pe^{rt}$ ✓✓

$\Rightarrow \frac{2000}{10} = \frac{1000}{10} e^{r3}$

$2 = e^{r3}$

$\frac{\ln 2}{3} = r$ ✓✓✓

$\Rightarrow A(t) = 10e^{(\frac{\ln 2}{3})t}$

$\frac{1000}{10} = \frac{10e}{10}^{(\frac{\ln 2}{3})t}$

$100 = e^{(\frac{\ln 2}{3})t}$ ✓✓

$\ln 100 = (\frac{\ln 2}{3})t$

$\frac{\ln 100}{(\frac{\ln 2}{3})} = t$

$19.93 = t$ ✓

$\Rightarrow$ $t = 20$ years $\Rightarrow$ 2030

4. (5 points) Solve the equation using any method you choose. Make sure to show your work (or explain what you did), and round to the nearest 0.001.

$$(1.00625)^{12t} = 2$$

$$\ln 1.00625^{12t} = \ln 2 \quad \checkmark$$

$$12t(\ln 1.00625) = \ln 2 \quad \checkmark$$

$$12t = \frac{\ln 2}{\ln 1.00625} \quad \checkmark$$

$$t = \frac{\left(\frac{\ln 2}{\ln 1.00625}\right)}{12} \quad \checkmark$$

$$\boxed{t = 9.271} \quad \checkmark$$

Extra Credit(2 points) What quadrant does $t = \frac{517\pi}{8}$ lie in? Explain how you got your answer.

$\frac{16\pi}{8} = 2\pi$

$16 \cdot 32 = 512$ ✓

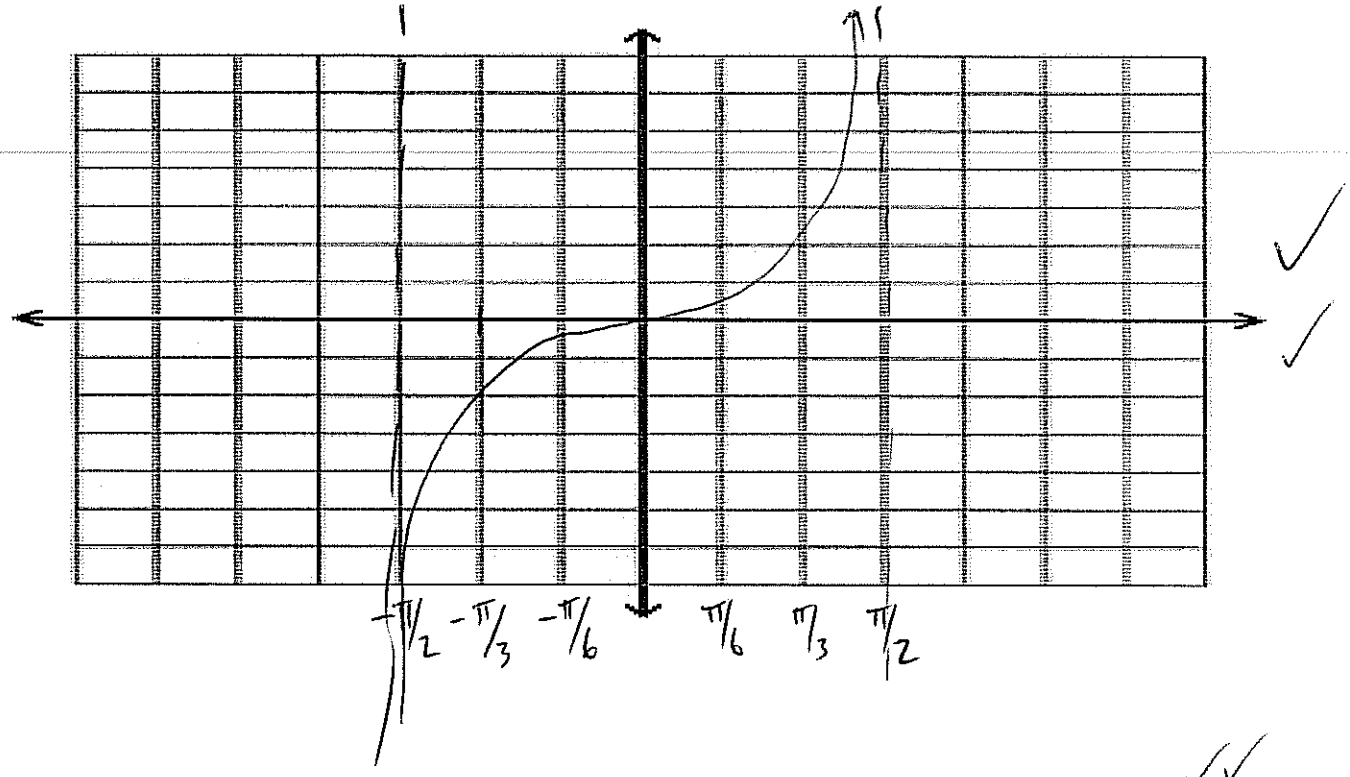
$\frac{517}{16} = 32.3125$

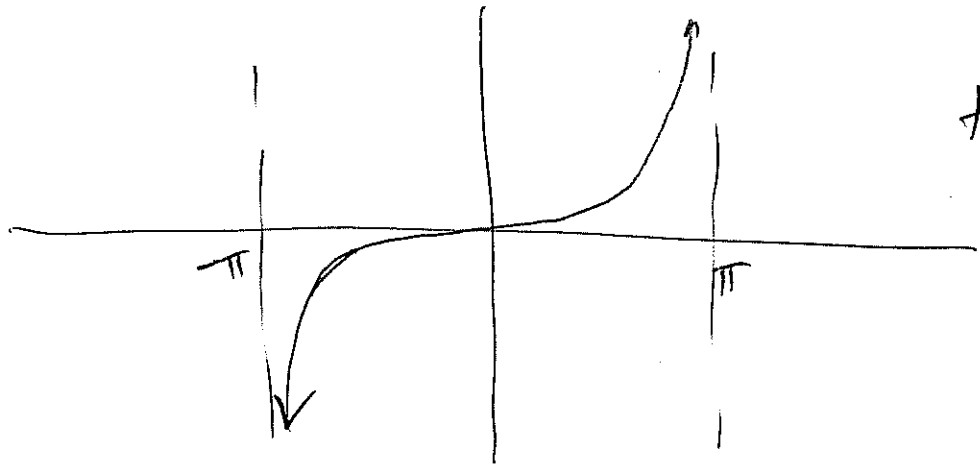
$\Rightarrow \frac{517\pi}{8} - \frac{512\pi}{8} = \frac{5\pi}{8} > \frac{\pi}{2}$

$\Rightarrow$ Quad II. ✓

Test C **No Calculator** Name: Key

5. (6 points) Sketch the graph of the function $y = \tan\frac{1}{2}\left(x - \frac{\pi}{3}\right)$. Make sure to label your axes with the appropriate numbers to indicate how the function is shifted.

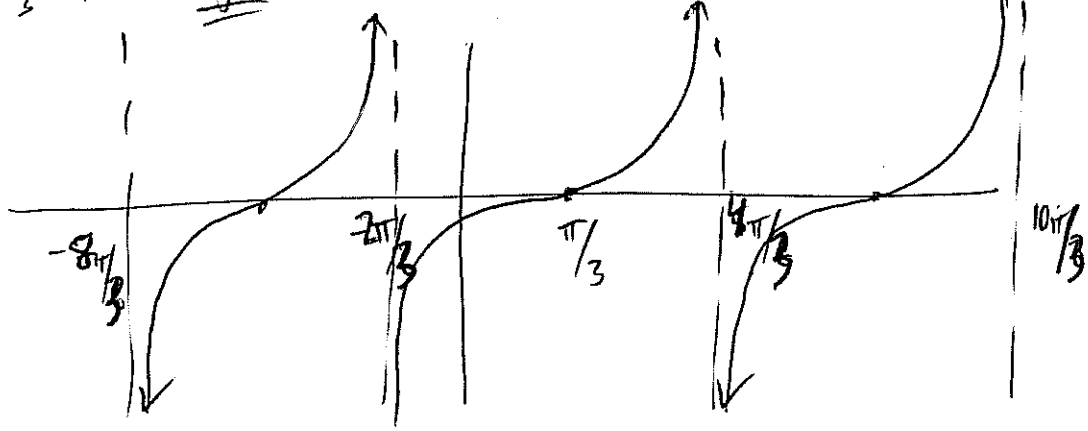




$\tan\frac{1}{2}x$

$\Rightarrow$ period $= \frac{\pi}{(\frac{1}{2})} = 2\pi$

Shift $\frac{\pi}{3}$ to the right.



briefly

6. (10 points) State if the following are True or False. If False, explain why.

(a) $\ln\left(\frac{x}{y}\right) = \frac{\ln x}{\ln y}$

False. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$, not $\frac{\ln x}{\ln y}$ 3

(b) $\log_7 7^7 = 7$

True b/c $\log_7 7^7 = 7 \log_7 7$

$= 7 \cdot 1$

$= 7$ 2

(c) $-\ln\left(\frac{1}{A}\right) = \ln A$

True b/c $-\ln\left(\frac{1}{A}\right) = -\ln\left(A^{-1}\right) = -1(-\ln A)$

$= \ln A$ 2

(d) $(\log_3 9)^x = x \log_3 9$

False $\log_3 9^x = x \log_3 9$ 3

and $(\log_3 9)^x \neq \log_3 9^x$

Note: $\log_3 9 = 2$ b/c $3^2 = 9$ and $2^x \neq x \cdot 2$

(up to 1.5 partial credit for work)

7. (5 points) Find all values of x that solve the equation. Make sure to show your work and explain your reasoning.

$$\log_2(x^2+4x+3)=3$$

$\Rightarrow x^2+4x+3=2^3=8$ ✓✓

$x^2+4x+3-8=0$ ✓

$x^2+4x-5=0$ ✓

$(x+5)(x-1)=0$ ✓

$\Rightarrow$ $x=-5$ or $x=1$

8

8. (6 points) Find the exact value (in its simplest form) for the following. If an expression does not exist, write DNE and explain why.

(a) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

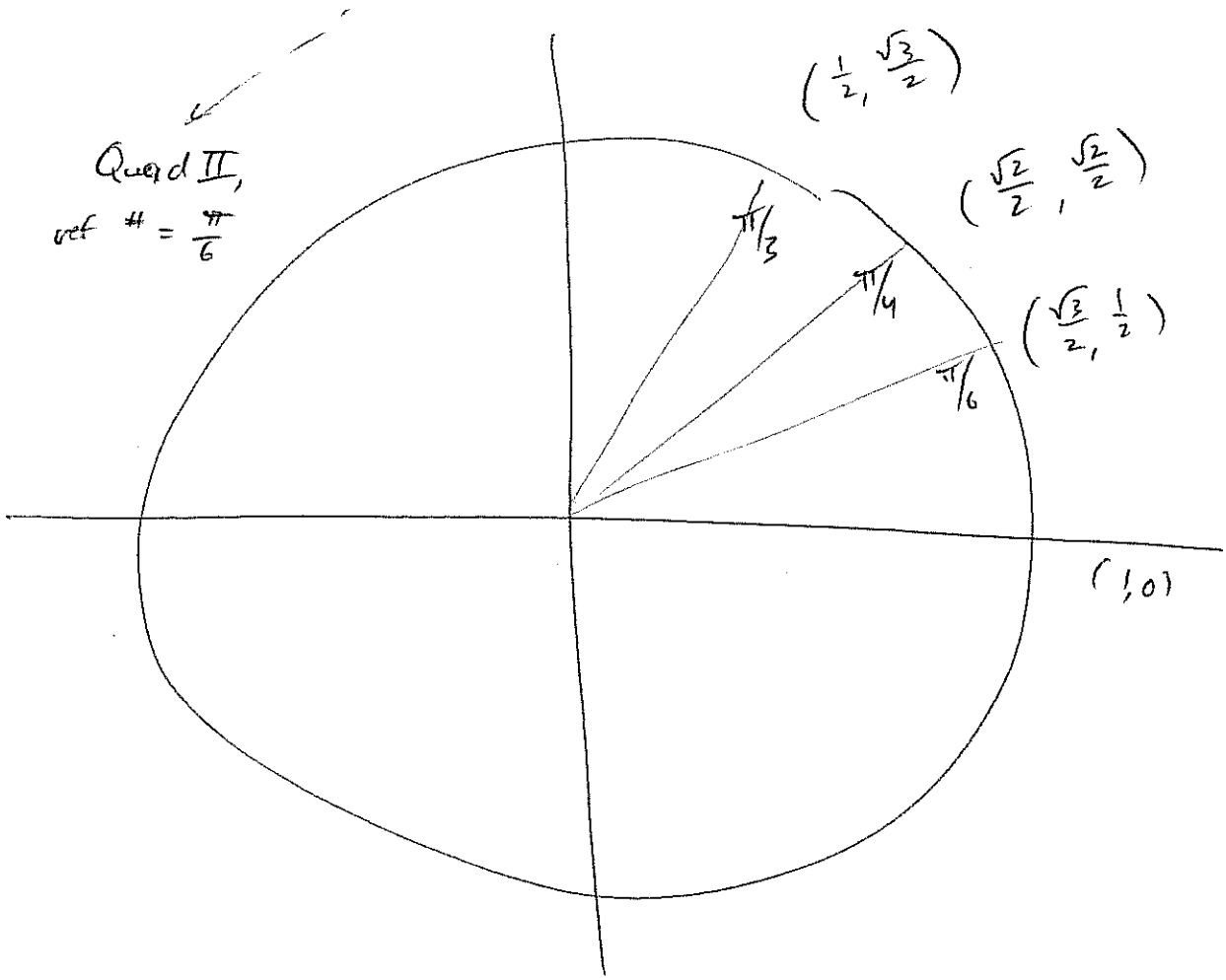
(b) $\cos\frac{19\pi}{6} = \cos\left(\frac{19\pi}{6} - \frac{12\pi}{6}\right) = \cos\left(\frac{7\pi}{6}\right)$ in Quad III, ref. # is $\pi/6$ $\Rightarrow \boxed{\cos\left(\frac{7\pi}{6}\right) = \frac{-\sqrt{3}}{2}}$

(c) $\tan\frac{9\pi}{4} = \tan\left(\frac{9\pi}{4} - \frac{8\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{1}$

(d) $\sec\frac{-2\pi}{3}$ is in Quad III, ref. # is $\frac{\pi}{3}$. $\cos\frac{-2\pi}{3} = \frac{-1}{2}$, $\sec\frac{-2\pi}{3} = \frac{1}{\cos\left(\frac{-2\pi}{3}\right)} = \frac{1}{\left(\frac{-1}{2}\right)} = \boxed{-2}$

(e) $\cot\frac{-16\pi}{4} = \cot -4\pi = \cot 0 = \frac{\cos 0}{\sin 0} = \frac{1}{0}$ $\boxed{\text{dne}}$ b/c cannot divide by zero.

(f) $\csc\frac{5\pi}{6} = \frac{1}{\sin\left(\frac{5\pi}{6}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = \boxed{2}$



9. (6 points) Given that $\csc t = \frac{5}{4}$ and t lies in Quadrant II, use trigonometric identities to find the value of $\tan t$.

$\csc t = \frac{5}{4} \Rightarrow \sin t = \frac{4}{5}$ ✓

$\sin^2 t + \cos^2 t = 1$ ✓

$\Rightarrow \left(\frac{4}{5}\right)^2 + \cos^2 t = 1$ ✓

$\frac{16}{25} + \cos^2 t = 1$

$\cos^2 t = 1 - \frac{16}{25}$ ✓

$\sqrt{\cos^2 t} = \sqrt{\frac{9}{25}}$

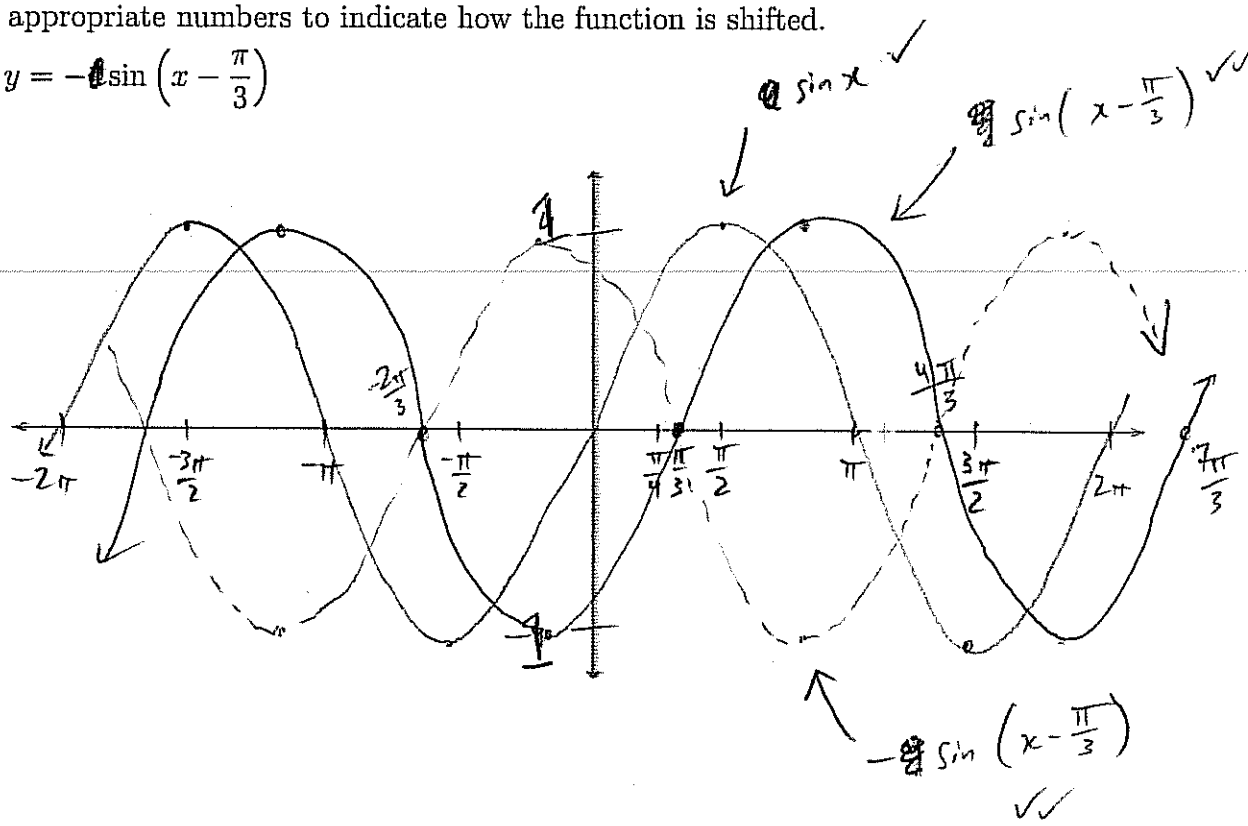
$\cos t = \pm \frac{3}{5}$ in Quad II $\Rightarrow \frac{-3}{5} = \cos t$ ✓

$\tan t = \frac{\sin t}{\cos t} = \frac{\frac{4}{5}}{\frac{-3}{5}} = \frac{4}{5} \cdot \frac{5}{-3} = \boxed{\frac{4}{-3}}$ ✓

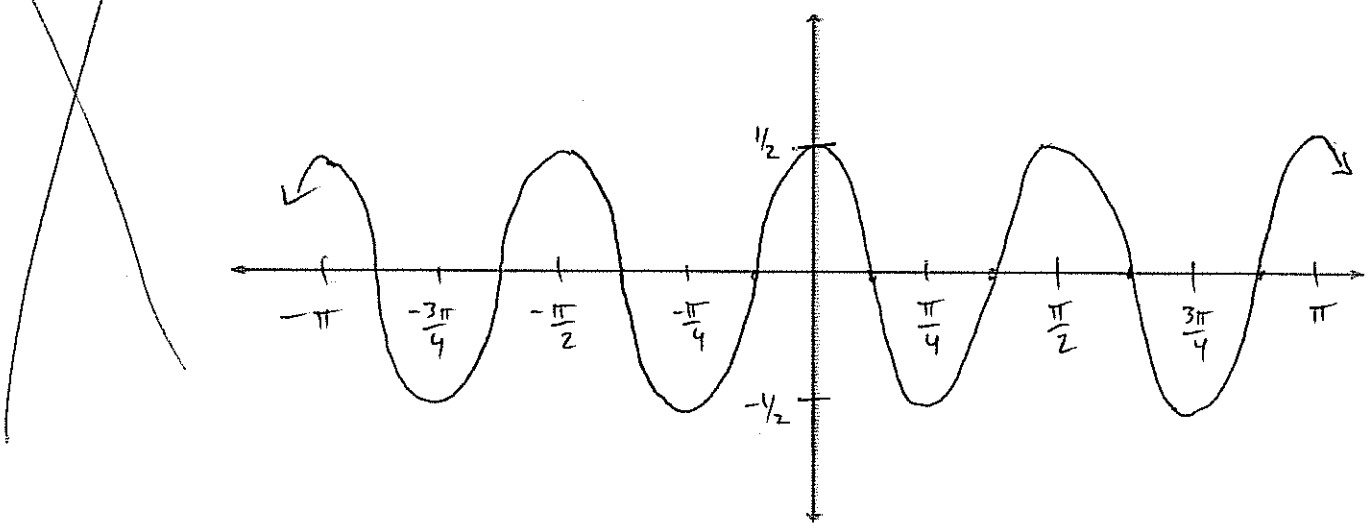
5

10. (10 points) Sketch graphs of the following functions. Make sure to label your axes with the appropriate numbers to indicate how the function is shifted.

(a) $y = -\sin\left(x - \frac{\pi}{3}\right)$



(b) $y = \frac{1}{2}\cos 4x \implies$ period $= \frac{2\pi}{4} = \frac{\pi}{2}$



Test 3D - MTH 1210

Dr. Graham-Squire, Fall 2013

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_____

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DIRECTIONS

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5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Make sure you sign the pledge.
8. Number of questions = 10. Total Points = 80.

1. (5 points) Solve the equation using any method you choose. Make sure to show your work (or explain what you did), and round to the nearest 0.001.

$$(1.00825)^{15t} = 3$$

$$\ln(1.00825)^{15t} = \ln 3$$

$$\Rightarrow \frac{(15t)\ln(1.00825)}{15\ln(1.00825)} = \frac{\ln 3}{15\ln 1.00825}$$

$$t = 8.914$$

2. (10 points) Use laws of logarithms to expand and simplify the expressions as much as possible.

(a) $\log_5 \sqrt{x\sqrt{y}}$ $= \log_5 (xy^{1/2})^{1/2} = \frac{1}{2}\log_5 (xy^{1/2})$

(b) $\ln\left(\frac{x^4y^3}{e^7}\right)$

$= \frac{1}{2}\left(\log_5 x + \log_5 y^{1/2}\right)$

$= \boxed{\frac{1}{2}\left(\log_5 x + \frac{1}{2}\log_5 y\right)}$

or

$\frac{1}{2}\log_5 x + \frac{1}{4}\log_5 y$

(b) $= \ln(x^4y^3) - \ln e^7$

$= \ln x^4 + \ln y^3 - 7$

$= \boxed{4\ln x + 3\ln y - 7}$

3. (12 points) The population of Fire-bellied Newts in the smaller of the Twin Lakes on Orcas island behaves according to the function

$$p(t) = \frac{4800}{3 + 9e^{-0.044t}}$$

where t is measured in years.

(a) What is the population initially?

(b) What is the population in the long run (as t goes to infinity)? Explain your reasoning.

(c) How many fire-bellied newts are there in the lake when t=10? Round to the nearest newt.

(d) When does the population reach 1000? Round to the nearest year.

(a) $t = 0 \Rightarrow \frac{4800}{3+9e^{-0.044(0)}} = \frac{4800}{3+9e^{0}} = \frac{4800}{12}$ $e^0 = 1$ 3

$= \boxed{400}$

(b) As $t \to \infty$, $e^{-0.044t} \to e^{-\infty} = \frac{1}{e^{\infty}} = 0$

$\Rightarrow p(t) \to \frac{4800}{3+9(0)} = \frac{4800}{3} = \boxed{1600}$ 3

(c) $p(10) = \frac{4800}{3+9e^{-0.044(10)}} = 545.68 = \boxed{546 \text{ newts}}$ 2

(d) $1000 = \frac{4800}{3+9e^{-0.044t}}$

$\Rightarrow (3+9e^{-0.044t})1000 = 4800$

$3+9e^{-0.044t} = \frac{4800}{1000}$

$9e^{-0.044t} = 4.8 - 3$

$e^{-0.044t} = \frac{1.8}{9}$

$\frac{\ln(0.2)}{-0.044} = t$ 4

$t = 36.58$

$\Rightarrow \boxed{t = 37 \text{ years}}$

4. (10 points) The rabbit population feeding off of Eddie's garden was 15 rabbits in the year 2009. In 2013, the number had grown to 30. Use an exponential growth model to calculate in what year will the population reach 800 rabbits. Round your answer to the nearest year.

Let 2009 be $t=0$. Then $P=15$ ✓✓

and $A=30$ when $t=4$

$A(t) = Pe^{rt}$ ✓✓

$\Rightarrow \quad 30 = 15e^{4r}$

$2 = e^{4r}$

$\frac{\ln 2}{4} = r$ ✓✓✓

$\Rightarrow \quad 800 = 15e^{(\frac{\ln 2}{4})t}$

$\frac{800}{15} = e^{(\frac{\ln 2}{4})t}$ ✓✓

$\frac{\ln(\frac{800}{15})}{(\frac{\ln 2}{4})} = t \approx 22.94$

$\Rightarrow$ 23 years ✓ $\Rightarrow$ 2032 ✓

Extra Credit(2 points) What quadrant does $t = \frac{465\pi}{8}$ lie in? Explain how you got your answer.

$\frac{16\pi}{8} = 2\pi \qquad \frac{465}{16} = 29.0625$ ✓

$29 \cdot 16\pi = 464\pi$ ✓

$\Rightarrow \frac{465\pi}{8} - \frac{464\pi}{8} = \frac{\pi}{8}$ is in Quad I

Test D **No Calculator** Name: Key

5. (8 points) Find the exact value (in its simplest form) for the following. If an expression does not exist, write DNE and explain why.

2 (a) $\sin\frac{\pi}{6}$ $= \boxed{\frac{1}{2}}$

2 (b) $\tan\frac{11\pi}{2} = \tan\frac{11\pi}{2} - \frac{4\pi}{2} = \frac{7\pi}{2} - \frac{4\pi}{2} = \frac{3\pi}{2}$. $\frac{3\pi}{2} \Rightarrow (0,-1)$

$\Rightarrow \tan\frac{11\pi}{2} = \frac{-1}{0}$ $\boxed{\text{dne}}$

2 (c) $\cot\frac{-15\pi}{4}$ $-\frac{15\pi}{4} + \frac{8\pi}{4} = -\frac{7\pi}{4} + \frac{8\pi}{4} = \frac{\pi}{4}$ $\cot\left(\frac{\pi}{4}\right) = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{1}$

2 (d) $\sec\frac{-2\pi}{3}$ θ has $\bar{t} = \frac{\pi}{3}$, in Quad III $\Rightarrow \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$$\sec\left(\frac{-2\pi}{3}\right) = \frac{1}{\left(-\frac{1}{2}\right)} = \boxed{-2}$$

6. (5 points) Find all values of x that solve the equation. Make sure to show your work and explain your reasoning.

$$\log_3(x^2 - 5x - 5) = 2$$

$\Rightarrow \quad x^2 - 5x - 5 = 3^2$ ✓✓

$x^2 - 5x - 5 = 9$ ✓

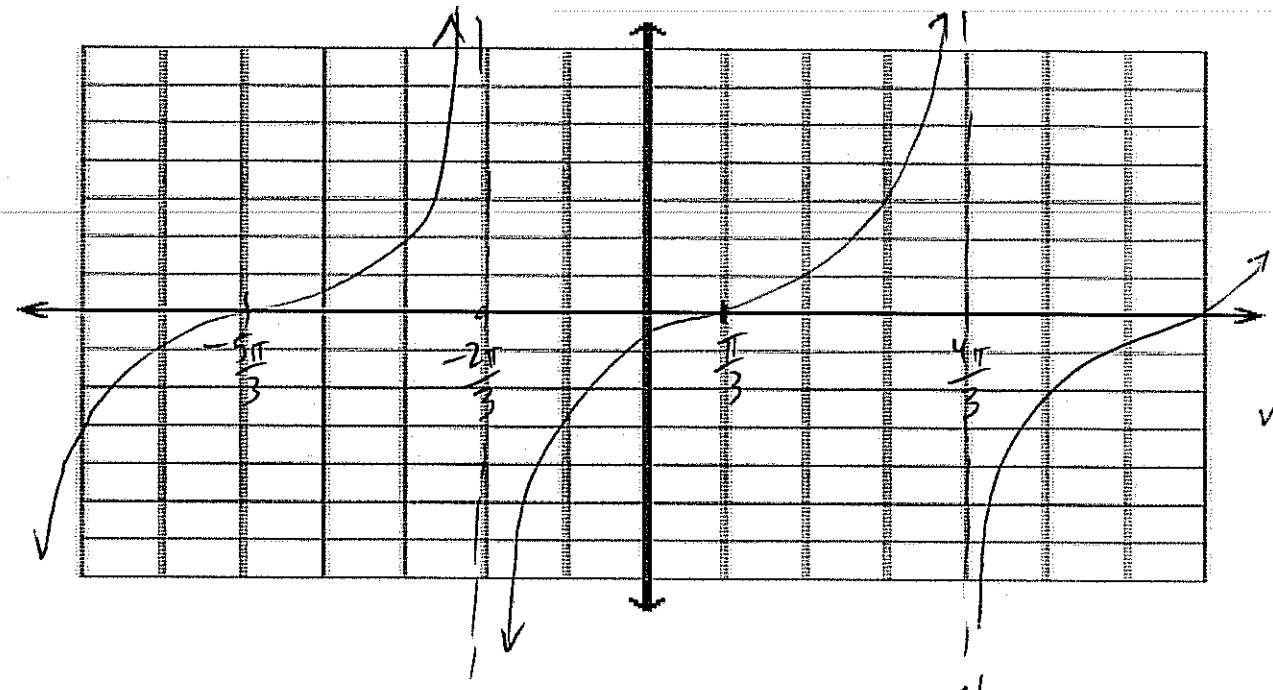
$x^2 - 5x - 14 = 0$ ✓

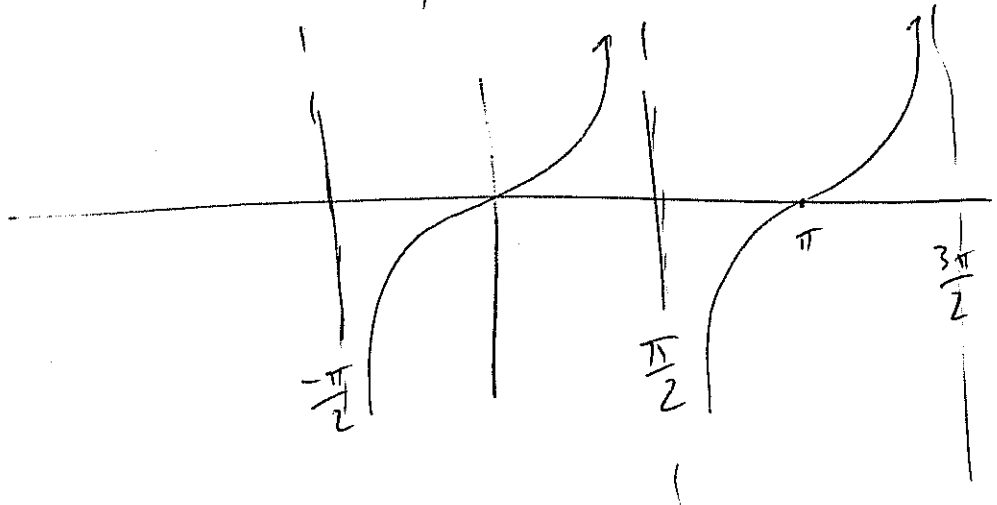
$(x - 7)(x + 2) = 0$ ✓

$\Rightarrow$ $\boxed{x = 7 \quad \text{or} \quad x = -2}$

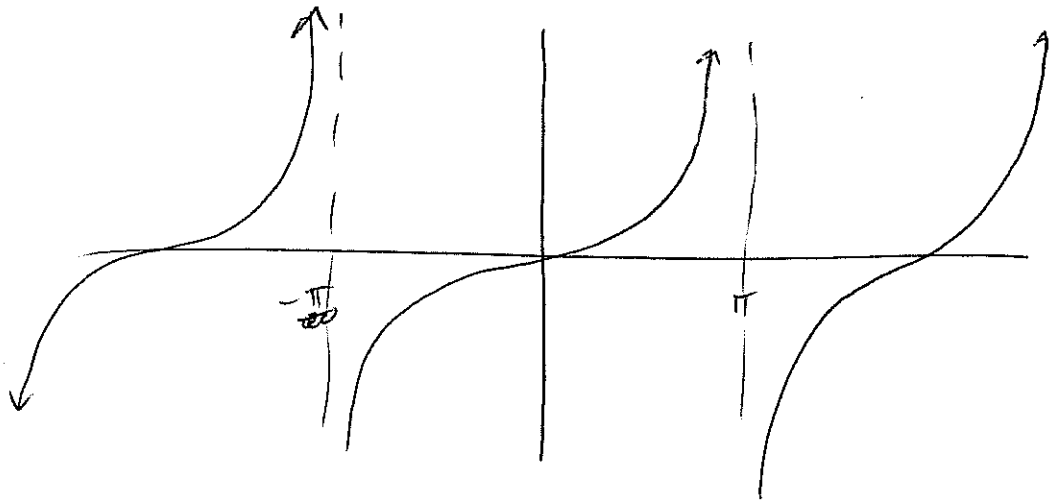
Both work in the original equation

7. (6 points) Sketch the graph of the function $y = \tan\frac{1}{2}\left(x - \frac{\pi}{3}\right)$. Make sure to label your axes with the appropriate numbers to indicate how the function is shifted.





tan x



tan $\frac{1}{2}x$

period $= \frac{\pi}{(\frac{1}{2})} = 2\pi$

8. (6 points) Given that $\sec t = \frac{-5}{4}$ and t lies in Quadrant III, use trigonometric identities to find the value of $\tan t$.

$$\frac{\sin^2 t}{\cos^2 t} + \frac{\cos^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$$

$$\tan^2 t + 1 = \sec^2 t \quad \checkmark \checkmark$$

$$\tan^2 t + 1 = \left(\frac{-5}{4}\right)^2 \quad \checkmark$$

$$\tan^2 t + 1 = \frac{25}{16}$$
$$-1 \qquad -1$$

$$\sqrt{\tan^2 t} = \sqrt{\frac{9}{16}} \quad \checkmark$$

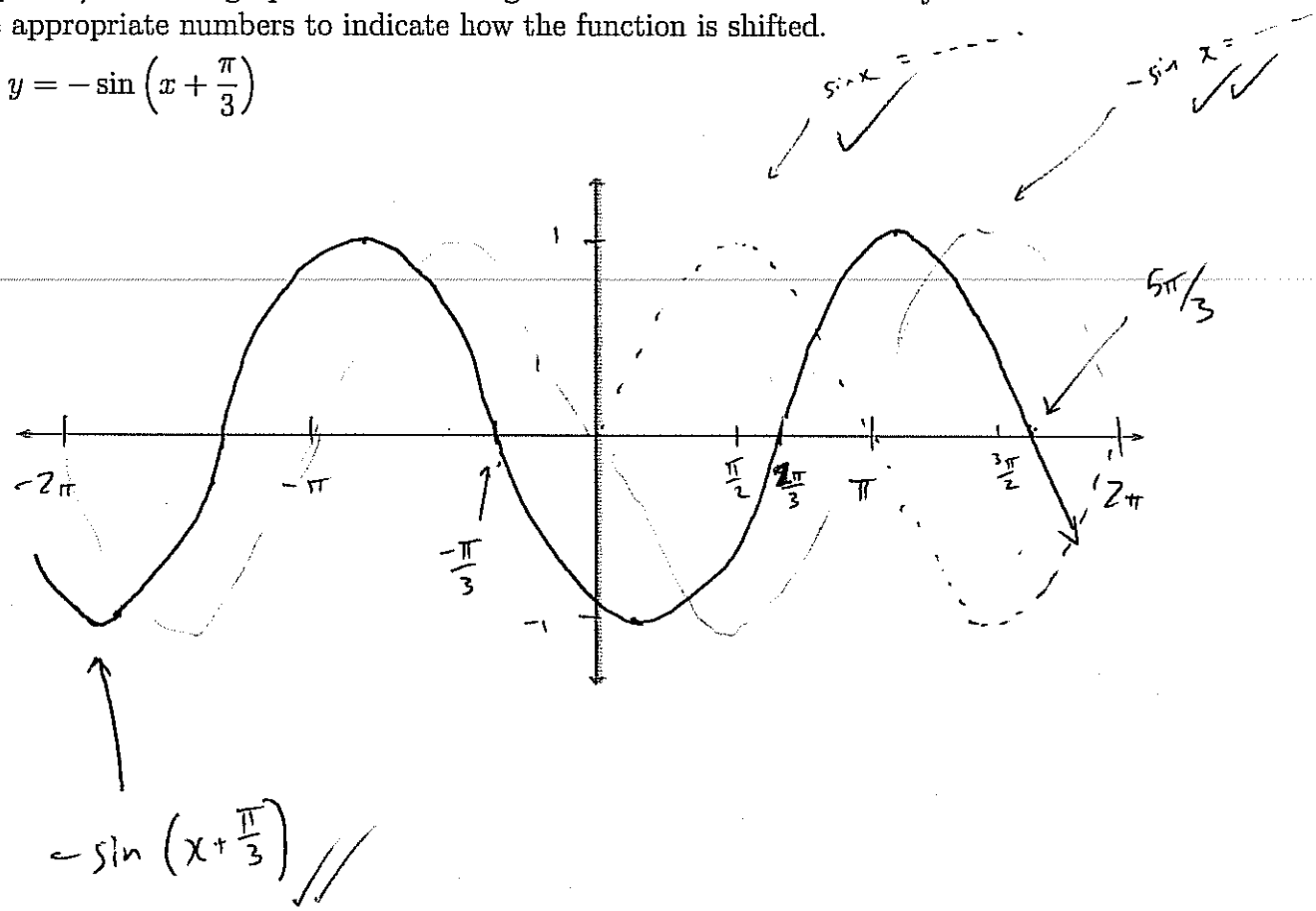
$$\tan t = \frac{\pm\sqrt{9}}{\sqrt{16}} = \pm\frac{3}{4} \quad \checkmark$$

in Quad III $\Rightarrow$ tan is + $\checkmark$

$$\boxed{\tan t = \frac{3}{4}}$$

9. (5 points) Sketch a graph of the following function. Make sure to label your axes with the appropriate numbers to indicate how the function is shifted.

(a) $y = -\sin\left(x + \frac{\pi}{3}\right)$



$-\sin\left(x + \frac{\pi}{3}\right)$ //

10. (10 points) State if the following are True or False. If False, explain why.

(a) $\log_5 5^5 = 5$

True b/c $\log_5 5^5 = 5(\log_5 5)$

$= 5(1)$

$= 5$

2

(b) $\ln\left(\frac{x}{y}\right) = \frac{\ln x}{\ln y}$

False $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$. Not division.

3

(c) $(\log_2 8)^x = x \log_2 8$

False $\log_2 8^x = x \log_2 8$, but $(\log_2 8)^x \neq \log_2 8^x$

3^x $\quad$ $x \cdot 3$

3

(d) $-\ln\left(\frac{1}{B}\right) = \ln B$

True: $-\ln\left(\frac{1}{B}\right) = -\ln\left(B^{-1}\right) = -(-\ln B) = \ln B$

2